

Chain of Majorana States from Superconducting Dirac Fermions at a Magnetic Domain Wall

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We study theoretically a strongly type-II *s*-wave superconducting state of two-dimensional Dirac fermions in proximity to a ferromagnet having in-plane magnetization. It is shown that a magnetic domain wall can host a chain of equally spaced vortices in the superconducting order parameter, each of which binds a Majorana-fermion state. The overlap integral of neighboring Majorana states is sensitive to the position of the chemical potential of the Dirac fermions. Thermal transport and scanning tunneling microscopy experiments to probe the Majorana fermions are discussed.

Majorana fermions are hypothetical particles that are their own antiparticles, as originally proposed to describe neutrinos [1]. Because of the non-Abelian statistics as Ising anyons in two spatial dimensions [2] and their potential application to a fault-tolerant quantum computation [2–4], the Majorana fermions have attracted revived interest in condensed-matter physics [5]. Theoretically, they appear as zero-energy states bound to vortices in topological superconductors [6], including $\nu = 5/2$ fractional quantum-Hall states [2], *s*-wave superconductors of two-dimensional (2D) Dirac fermions [7], and chiral *p*-wave superconductors [6, 8]. However, it remains a challenge to realize and manipulate Majorana fermions in a practically detectable and controllable manner.

Recently, there have been several proposals for realizing Majorana fermions as the edge modes at heterostructure interfaces. It has been argued that the superconductivity induced by the proximity effect on the surface of a three-dimensional (3D) topological insulator (TI) [9–11] can host Majorana modes propagating along the edge of the surface. When the superconducting region is surrounded by two magnets having opposite out-of-plane magnetizations, two charge-neutral Majorana edge modes are combined into a charged Dirac fermion, which can be probed with charge transport. A Rashba-semiconductor thin film sandwiched by an insulating magnet having an out-of-plane magnetization and an *s*-wave superconductor has also been proposed to produce a chiral Majorana edge mode [12–15].

In this Letter we show theoretically that a magnetic domain wall in an insulating ferromagnet attached to a 2D strongly type-II *s*-wave Dirac-fermion superconductor can produce a chain of vortices [Fig. 1(a)], each accommodating a Majorana zero mode [3], when the magnetization points to the out-of-plane direction at the head-to-head magnetic domain wall. A hybridization of Majorana fermions at neighboring vortices can be tuned by the chemical potential of the Dirac-fermion superconductor, which can be probed with thermal transport and scanning tunneling microscopy (STM) experiments. We also propose two heterostructures that can realize such

electronic states: (A) an interface of an insulating ferromagnetic thin film and a bulk *s*-wave superconductor whose normal state is a 3D TI [Fig. 1(b)] and (B) a device structure with stacked *s*-wave superconductor, Rashba semiconductor, and insulating magnetic film [Fig. 1(c)].

We consider a 2D continuum BCS-Dirac Hamiltonian with a single Dirac cone in the single-particle dispersion,

$$H_0 = \int d^2\mathbf{x} \psi^\dagger(\mathbf{x}) \{ [iv\mathbf{D} \times \mathbf{e}_3 - \mathbf{B}(\mathbf{x})] \cdot \boldsymbol{\sigma} - \mu\sigma_0 \} \psi(\mathbf{x}) + \int d^2\mathbf{x} \left\{ \Delta(\mathbf{x})\psi^\dagger(\mathbf{x})i\sigma_2[\psi^\dagger(\mathbf{x})]^T + \text{H.c.} \right\}, \quad (1)$$

under the Zeeman field $\mathbf{B}(\mathbf{x})$, i.e., an exchange field coming from a ferromagnet. Here, v is the Dirac-fermion velocity, μ the chemical potential, and the covariant derivative $\mathbf{D} = \nabla - ie\mathbf{A}$ with \mathbf{A} being the vector potential. The spinors $\psi^\dagger(\mathbf{x}) = (c_{\mathbf{x}\uparrow}^\dagger, c_{\mathbf{x}\downarrow}^\dagger)$ and $\psi(\mathbf{x}) = (c_{\mathbf{x}\uparrow}, c_{\mathbf{x}\downarrow})$ create and annihilate fermions at position \mathbf{x} with charge e , respectively. We use $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ for the Pauli matrices as well as σ_0 for the 2×2 unit matrix. The unit vector \mathbf{e}_3 is normal to the 2D plane, while $\mathbf{x} \equiv x_1\mathbf{e}_1 + x_2\mathbf{e}_2$ lies in the plane. In the absence of the magnetic field $\mathbf{B}(\mathbf{x}) = 0$, the superconducting order parameter $\Delta(\mathbf{x})$ is assumed to take a uniform constant

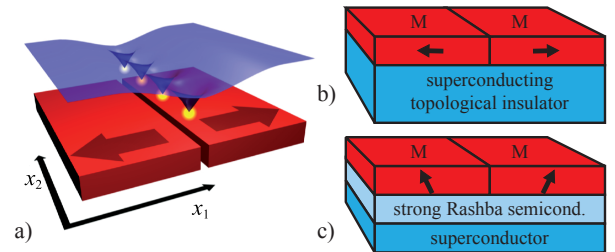


FIG. 1. (Color online) (a) Schematic picture of the modulus of the superconducting order parameter (thin surface) in either of the two proposed heterostructures shown in (b) and (c). Along the boundary between two domains of a ferromagnetic insulator (M) with opposite in-plane magnetization (bold arrows), a chain of vortices emerges. A single Majorana fermion is bound to each vortex (dots).

Δ_0 , which we choose real and positive. Henceforth, we assume strong type-II superconductivity so that the orbital coupling to the vector potential \mathbf{A} can be ignored in the analysis of $\Delta(\mathbf{x})$.

Here, we introduce a head-to-head domain wall that extends along the \mathbf{e}_2 -direction [Fig. 2(a)]. Away from the domain wall, the Zeeman field is given by $\mathbf{B} \rightarrow \pm B\mathbf{e}_1$ for $x_1 \rightarrow \pm\infty$, respectively. Therefore, it can be approximated with a uniform in-plane Zeeman field $\pm B\mathbf{e}_1$. This shifts the Dirac cone and therefore causes a finite center-of-mass momentum $\mathbf{q} = \pm q\mathbf{e}_2$ of the spin-singlet Cooper pairs with $q = 2B/v$ [16]. As a consequence, the superconducting order parameter Δ will acquire a spatial modulation $\Delta(\mathbf{x}) = \Delta_0 e^{iq \cdot \mathbf{x}}$ for $x_1 \rightarrow \pm\infty$.

Alternatively, the spatial modulation of $\Delta(\mathbf{x})$ by the in-plane Zeeman field can be described using the Ginzburg-Landau (GL) free energy,

$$F = \int d^2\mathbf{x} [\alpha|\Delta(\mathbf{x})|^2 + \gamma|\mathcal{D}_\Delta\Delta(\mathbf{x})|^2 + u|\Delta(\mathbf{x})|^4], \quad (2)$$

where $\mathcal{D}_\Delta = \nabla - 2i(e\mathbf{A} + v^{-1}\mathbf{A})$ and we introduced the real parameters $\alpha, \gamma > 0$, and $u > 0$. The linear coupling to the Zeeman field, written in terms of the fictitious vector potential $\mathcal{A} := \mathbf{e}_3 \times \mathbf{B}$, is allowed due to the broken mirror symmetry about the 2D plane. Away from the

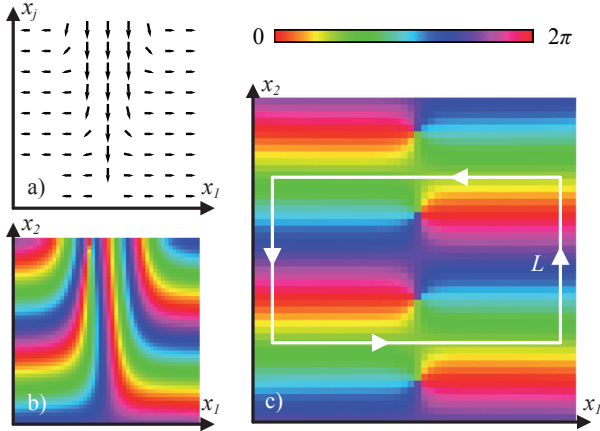


FIG. 2. (Color online) (a) Magnetic moment of the ferromagnet around the domain walls of types (i) and (ii). This structure continues in the other spatial direction. (b) and (c) Color maps of the phase of the superconducting order parameter $\Delta(\mathbf{x})$ in our numerical solutions of the GL model Eq. (2) on a grid of 50×50 sites for the types (i) and (ii) of head-to-head domain walls, respectively. They have been obtained at the criticality by ignoring the quartic term. In the \mathbf{e}_2 direction, open boundary conditions and periodic boundary conditions were chosen for cases (i) and (ii), respectively. Fixed boundary conditions resembling the solutions deep inside each domain were imposed in the \mathbf{e}_1 direction. The domain-wall parameters were set to $l = 6$ lattice sites and $s = 1/4$ while B/v was chosen such that two periods of the phase modulation fit into the grid. A variation in the model parameters l , s and $|B/v|$ does not change the result qualitatively.

domain wall, the mean-field solution in the strongly type-II limit can indeed be obtained as $\Delta(\mathbf{x}) = \Delta_0 e^{iq \cdot \mathbf{x}}$ with $\Delta_0 = \sqrt{|\alpha|/2u}$ for $\alpha < 0$. We employ this GL theory to determine a spatial profile of $\Delta(\mathbf{x})$ in the domain-wall region where the Zeeman field \mathbf{B} varies with \mathbf{x} .

In fact, the profile of $\Delta(\mathbf{x})$ strongly depends on whether the magnetic moment of the attached ferromagnetic film changes its direction along the domain wall (i) within the plane as in stripes of magnetic thin films [17] or (ii) towards the out-of-plane direction as in magnetic thin films with large magnetic anisotropy [18]. The magnetic moment near these domain walls can be modeled as a divergence-free function,

$$\mathbf{M}(\mathbf{x}, x_3) = B \sum_{\lambda=\pm s} \frac{\mathbf{e}_1 - \lambda \mathbf{e}_j}{2} \tanh\left(\frac{\lambda x_1 + x_j}{\lambda l}\right), \quad (3)$$

with $j = 2$ for the case (i) and $j = 3$ for the case (ii), where l and s control the characteristic width of the domain wall. We assume that the Zeeman field \mathbf{B} acting on the Dirac-fermion superconductor at the interface $x_3 = 0$ is given by $\mathbf{B}(\mathbf{x}) = \mathbf{M}(\mathbf{x}, 0)$. A key observation here is that the out-of-plane component $B_3(\mathbf{x})$ vanishes. Then, this in-plane Zeeman field induces a fictitious magnetic field defined by $\mathcal{B} := \nabla \times \mathcal{A}$, which becomes $\mathcal{B} = 0$ and $\mathcal{B} = \mathbf{e}_3(B/l) \text{sech}^2(x_1/l)$ for cases (i) and (ii), respectively. In analogy to the vortex-lattice phase of a type-II superconductor, we would thus expect to obtain vortices at the domain wall only for case (ii). This is confirmed by the numerical solutions of the phase of the superconducting order parameter $\Delta(\mathbf{x})$ for the cases (i) and (ii), presented in Figs. 2(b) and (c), respectively. For case (i), the contour of the phase of $\Delta(\mathbf{x})$ follows the magnetic flux \mathbf{B} and hence no vortices emerge. For case (ii), a chain of vortices in $\Delta(\mathbf{x})$ appears along the domain wall. All the vortices have the same winding number $+1$ or -1 , where the sign is determined by the sign of B/v .

The emergence of the vortices in case (ii) can also be understood from the integration of the phase of $\Delta(\mathbf{x})$ deep inside the two domains, i.e., along the contour drawn in Fig. 2(c). Provided $\Delta(\mathbf{x})$ does not vanish along the contour, the flux enclosed by this contour takes an integer multiple of the superconducting flux quantum Φ_0 , that monotonically changes by 1 each time the contour length along the domain wall is increased by π/q . It indicates that the intervortex distance is given by π/q . In Fig. 2(c) we observe that $\Delta(\mathbf{x})$ changes sign when translating it by the separation of neighboring vortices $\mathbf{x}_q = (0, \pi/q)$, i.e., $\Delta(\mathbf{x}) = -\Delta(\mathbf{x} + \mathbf{x}_q)$, $\forall \mathbf{x}$.

Having determined the spatial structure of the order parameter, we will now discuss how Majorana fermions emerge. An *isolated* vortex in the superconducting order parameter comprises a Majorana fermion as a bound state at precisely zero energy. However, in the vortex chain, these bound states can potentially couple to each other and form an energy band around zero energy. To

study this, we will construct an effective tight-binding model for the Majorana fermions in the vortex chain.

First, let us consider an isolated vortex located at $\mathbf{x} = 0$ in the vicinity of a domain wall of type (ii), i.e., at $|x_1| \ll l$, in the strongly type-II limit, by assuming $|\mu| \ll \Delta_0$ and $|B| \ll \Delta_0$. We also assume that the vortex core radius, which is given by the coherence length $\xi = v/\Delta_0$ from the analogy to conventional type-II superconductors, is much shorter than the domain-wall thickness l ; $\xi \ll l$. Then, the problem is reduced to a Bogoliubov-de Gennes Hamiltonian $H_\Delta := \int d^2\mathbf{x} \Psi^\dagger(\mathbf{x}) \mathcal{H}_\Delta \Psi(\mathbf{x})$, where

$$\mathcal{H}_\Delta := \begin{pmatrix} -\mu & vk & \Delta & 0 \\ vk^\dagger & -\mu & 0 & \Delta \\ \Delta^* & 0 & \mu & -vk \\ 0 & \Delta^* & -vk^\dagger & \mu \end{pmatrix}. \quad (4)$$

We used the bispinors $\Psi^\dagger(\mathbf{x}) = (\psi^\dagger(\mathbf{x}), -i\psi^T(\mathbf{x})\sigma_2)$ and the complex momentum operators $k := -\partial_1 + i\partial_2$ and $k^\dagger = \partial_1 + i\partial_2$. When $B > 0$, the order parameter $\Delta(\mathbf{x})$ has the form $\Delta_\pm(\mathbf{x}) = \pm \tilde{\Delta}(|\mathbf{x}|)z/|\mathbf{x}|$ for $|\mathbf{x}| \ll \xi$, where $z = x_1 + ix_2$ and $\tilde{\Delta}(x)$ is a real non-negative function with $\tilde{\Delta}(0) = 0$. With the sign \pm we allow for two different choices of the gauge for $\Delta(\mathbf{x})$. A zero mode $\Psi_\pm(\mathbf{x})$ bound to the vortex is the solution to $\mathcal{H}_{\Delta_\pm} \Psi_\pm(\mathbf{x}) = 0$, which is normalizable, i.e., $\int d^2\mathbf{x} \Psi_\pm^\dagger(\mathbf{x}) \Psi_\pm(\mathbf{x}) < \infty$. To linear order in μ , it is given by

$$\Psi_\pm(\mathbf{x}) = \frac{1}{\sqrt{\mathcal{N}}} \begin{pmatrix} 1 \\ -\frac{\mu}{2v}z \\ \pm \frac{\mu}{2v}z^* \\ \pm 1 \end{pmatrix} \exp \left[-\frac{1}{v} \int_0^{|\mathbf{x}|} dx \tilde{\Delta}(x) \right], \quad (5)$$

where \mathcal{N} is the normalization factor [19].

We turn to the case where $\Delta(\mathbf{x})$ includes the vortex chain instead of an isolated vortex. The vortices are centered at $\mathbf{x} = (n - \frac{1}{2})\mathbf{x}_q$, ($n = 0, \pm 1, \pm 2, \dots$). The overlap integral t of the Majorana modes bound to two neighboring vortices can be written in terms of two solutions for the isolated vortex, Ψ_+ and Ψ_- , as

$$t := \int d^2\mathbf{x} \Psi_+^\dagger \left(\mathbf{x} + \frac{\mathbf{x}_q}{2} \right) \mathcal{H}_\Delta \Psi_- \left(\mathbf{x} - \frac{\mathbf{x}_q}{2} \right). \quad (6)$$

In our case of $|B| \ll \Delta_0$, which yields $\xi \ll \pi/|q|$, we can approximate $\Delta = i\Delta_0 [\tilde{\Delta}(x) = \Delta_0]$ in Eq. (6). We obtain

$$t = C\mu \left(\frac{\Delta_0}{|B|} \right)^{3/2} \exp \left(-\frac{\pi\Delta_0}{2|B|} \right) + \mathcal{O}(\mu^2/\Delta_0) \quad (7)$$

as the leading asymptotic behavior in $|\mu|/\Delta_0 \ll 1$, where C is a dimensionless constant of order one that depends on the spatial structure of $\Delta(\mathbf{x})$. We note that the inclusion of the in-plane component of the Zeeman field in Eq. (4) for the calculation of t does not change the leading asymptotic behavior. The above example demonstrates

that we can create a chain of Majorana fermions whose transfer integral t is controlled by μ as well as $|B|/\Delta_0$.

The vanishing of the hybridization and the emergence of many Majorana zero modes at $\mu = 0$ can also be explained in terms of Weinberg's index theorem on the number of fermion zero modes [20, 21]. Namely, there exist at least $|n|$ zero-energy states for our Hamiltonian Eq. (4) at $\mu = 0$, where $n := \frac{1}{2\pi} \oint_{\mathcal{C}} d\mathbf{x} \cdot \nabla \arg \Delta$ is the number of vortices enclosed by a loop \mathcal{C} . This property survives, when the in-plane Zeeman field $\mathbf{B}(\mathbf{x}) = \mathbf{M}(\mathbf{x}, 0)$ is restored in the Hamiltonian (4), where \mathbf{M} is in the form of Eq. (3) with $j = 3$. Then, \mathbf{B} at $|x_1| \rightarrow \infty$ can be absorbed by a gauge transformation resulting in an exponentially decaying fictitious vector potential \mathcal{A} , and hence the condition for the theorem [20] is satisfied. In contrast, if there remains a non-negligible out-of-plane Zeeman field $B_3(\mathbf{x})$ unlike the above case, it couples the Majorana fermions and lifts their degeneracy.

A tight-binding chain of Majorana fermions can be formulated in general with the real scalar Majorana fields ϕ_j by the Hamiltonian,

$$H_{\text{eff}} = i \sum_j (t + \delta t_j) \phi_j \phi_{j+1}. \quad (8)$$

Here, j labels the vortices along the chain and δt_j are random fluctuations of the hopping amplitude caused by spatial inhomogeneity of μ . A similar model was introduced by Kitaev in Ref. 22 and has recently been studied in Ref. 23. Since the Majorana fermions are charge neutral, they do not contribute to the electric transport but to the thermal transport and the specific heat. The low-temperature one-dimensional thermal conductivity κ due to the Majorana fermions is obtained within the semiclassical transport theory as $\kappa/T = \pi^2 k_B^2 |t| \tau / (3|q|)$. Here k_B is the Boltzmann constant and τ the elastic scattering time associated with the backscattering due to the fluctuations δt_j . The presence of the Majorana fermions can be confirmed by the measurement of their T -linear contribution to κ , with the characteristic V-shaped dependence of κ on $|\mu|$ across the Dirac nodal point.

An alternative direct way of detecting the Majorana-fermion chain would be an STM experiment on the superconducting surface. This requires a careful fabrication of a spatial gap in the form of a strip separating two oppositely magnetized ferromagnets, to allow the STM tip to probe the superconducting surface.

For these measurements, the temperature should be low enough to suppress contributions from other midgap states bound to the vortices. The second-lowest-energy midgap states appear at $|E_1| = \sqrt{3}\Delta_0/2$ in the limit $\xi|q| \rightarrow 0$ and at $\mu = 0$ [24]. This leads to the condition $k_B T \ll \Delta_0$. The suppression of phonon and magnon contributions to κ also requires low temperature: The phonon contribution can be separated by its cubic temperature dependence. An insulating ferromagnetic thin

film with a large magnon gap can be used to suppress magnon contributions.

We will now discuss a possible experimental realization of our proposal. If a 3D TI becomes superconducting upon cooling, the surface Dirac fermions associated with the 3D TI acquire an energy gap. Then, this may be described as an s -wave superconductivity of Dirac fermions hosting a single Dirac cone in the normal state. When an insulating ferromagnetic thin film is deposited on this surface, it will provide an ideal laboratory for our theoretical model. Candidate materials for the 3D TI with superconducting instability include $\text{Cu}_x\text{Bi}_2\text{Se}_3$ which becomes a strongly type-II superconductor by doping Cu into the parent 3D TI Bi_2Se_3 [25, 26], and the superconductor LaPtBi which has been predicted theoretically as a 3D TI [27]. The chemical potential needs to be carefully controlled by applying a gate voltage or chemical doping. Insulating ferromagnetic thin films of spinel compounds CdCr_2Se_4 and CdCr_2S_4 [28] could be deposited on the surface. In particular, slightly off-stoichiometric compounds are useful for inducing appreciable magnetic anisotropy in the parent compounds [29] and thus suppressing the magnon contribution to the thermal conductance. A magnetic structure needs to be carefully examined with Lorentz microscope. Typically, the domain structures are filamentary, thus creating several parallel chains of Majorana fermions. Then, their contributions to κ may sum up to a larger signal. With realistic model parameters $v \approx 0.3 \text{ eV nm}$ for the 3D TI Bi_2Se_3 [30], and $\Delta_0 \approx 0.5 \text{ meV}$, the spatial separation of two neighboring vortices is given by $\pi/q = \pi v/2B \approx 5 \mu\text{m}$, provided $B \approx 0.1 \text{ meV}$. The angle-resolved photoemission spectroscopy measurement for $\text{Cu}_x\text{Bi}_2\text{Se}_3$ at the optimal doping $x = 0.12$ shows $\mu \approx 0.5 \text{ eV}$, which is about 0.25 eV above the bottom of the 3D conduction band [26] and clearly in the limit $|\mu| \gg \Delta_0$. In this case the transfer integral t is expected to be much larger than the value estimated from Eq. (7) in the opposite limit, $|t| \approx 4 \times 10^{-3} |\mu|$.

Lastly, we propose an alternative way of creating a chain of Majorana fermions by using a heterostructure based on a Rashba-semiconductor, i.e., InAs, sandwiched by an s -wave superconductor and an insulating ferromagnet with a domain wall [Fig. 1(c)]. We apply a magnetic field parallel to \mathbf{e}_3 to tilt the magnetization deep inside the two domains of the ferromagnetic insulator, yielding the effective Zeeman field in the semiconductor, $\mathbf{B} \rightarrow \pm B\mathbf{e}_1 + B_3\mathbf{e}_3$, for $x_1 \rightarrow \pm\infty$, where the domain wall extends along the \mathbf{e}_2 -direction. The chemical potential needs to be tuned closed to the Γ -point energy level, so that B_3 gaps out the smaller of the two spin-orbit split Fermi surfaces. By linearizing the dispersion near the remaining Fermi surface, the effective noninteracting Hamiltonian takes the same form as Eq. (1). This can therefore produce a similar Majorana-fermion chain. In this case, however, one cannot largely control its energy bandwidth, since the chemical potential for the original

electrons is always away from the Dirac nodal point.

In conclusion, we have shown that a chain of Majorana fermions can be created along the domain wall of a ferromagnetic insulator that is coupled to a superconductor with a single Fermi surface of helical electrons. In the search for devices that allow the manipulation of Majorana fermions as building blocks for a topological quantum computer, the proposed structure could in principle serve as a one-dimensional circuit path. In particular, the possibility to adjust the coupling between neighboring Majorana fermions by changing the chemical potential can be beneficial in this context.

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